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**THE APPLICATION OF PROBABILITY
CALCULATIONS FOR BIRD-AIRCRAFT STRIKE
ANALYSES AND PREDICTIONS USING RADAR**

George E. Meyer

**Air Force Weapons Laboratory
Kirtland Air Force Base, New Mexico**

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A theoretical development is given for the calculation of binomial probability distribution functions for assessing the risk of bird hazards to aircraft using radar. A set of airspace cells is defined by beam geometry and pulse width and a given aircraft flight path, each with a determined number of birds. Each distribution function can be studied to determine the maximum risk and corresponding number of birds involved. The accumulative probability of bird strikes over an entire route can be determined by calculating the union of discrete cell probability sets.		

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SECTION I

INTRODUCTION

The basic goal of the use of radar detection and avoidance of bird-aircraft collisions is to aid aircraft in avoiding birds by locating and assessing bird hazard levels and by providing a basis for decision-making for the next course of action. Air traffic controllers (ATC) use radar for regulating air traffic. Radar has also played a significant role in weather forecasting and, more recently, by ornithologists for studying bird movements. Radar could be used for continually monitoring and evaluating bird hazards. This methodology falls into the more general category of value engineering and risk determination which asks the basic question: What engineering or procedural changes are necessary for increased safety?

The destructive potential of bird-aircraft collisions has been very well documented in the literature and from accident records. The impact forces in such collisions have been widely discussed. Development in windscreen and jet engine components hardening has begun, even under the most stringent design requirements of today's sophisticated high-performance aircraft. Yet, until impact load requirements are met or new, lightweight, high-strength materials are found, solutions not expected for many years to come, procedural changes may offer the best current help for the problem.

The stochastic or probabilistic nature of the bird accident problem has been generally recognized. Many factors influence the problem; these have been categorized as mission or operational and biological.

Because of its probabilistic nature, the underlying problem is the lack of a complete instrumentation package which gives high levels of precision in determining altitudes, times, and locations of birds with respect to a locally flying aircraft. Radar is the best instrumentation package to date. High levels of confidence of detection and essential data (minimization of false alarm rates) are necessary for accurate hazard warnings needed to facilitate decision making.

It is the purpose of this technical report to expand on the use of probability theory to aid the reduction of bird strikes and to correct what appear to be certain misconceptions about its use in the bird strike literature. This will be

achieved by applying elementary properties of probability sets to the development of a basic model which can be applied to radar detection and avoidance.

Section 1

INTRODUCTION

The basic goal of this report is to develop a model for radar detection and avoidance. The model is based on the assumption that the radar system is a Markov process. The model is developed in two parts. The first part is a description of the radar system. The second part is a description of the avoidance system. The model is developed in two parts. The first part is a description of the radar system. The second part is a description of the avoidance system. The model is developed in two parts. The first part is a description of the radar system. The second part is a description of the avoidance system.

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SECTION II

BACKGROUND

Probability theory can be successfully applied to phenomena which cannot be precisely predicted. A card drawn blindly from a deck of cards or noise generated in an electrical system are common examples. Events of absolute certainty are assigned a probability of one, while an impossible event is given a probability of zero. Those events that occur with intermediate values are often of most concern. A basic question that is asked any experiment in research is what can be said about the possible outcome generated by an incomplete set of data? It must always be recognized that sufficient data must be available to support the simplest of conclusions. Most real world problems fall into the above category. Information theory and cybernetics are strongly related to probabilities.

Air Force research has not successfully addressed probabilities for bird strike phenomena, as has been attempted in the literature. Bird strikes are often considered random phenomena, yet no formalism of probabilities has been achieved. The problem on the grand Air Force scale involves a complexity of factors. If the grand scale problem were successfully modeled, the model would stagger the imagination.

A much more useful consideration involves the tracking of birds and aircraft using radar and arriving at useful conclusions for detection and avoidance. Any region under radar surveillance is defined by the elevational width of the beam and the angle of sweep of the antenna. Such a region is three dimensional in nature, and its outer limit or maximum range is determined by the threshold detection limit. Detection limits in themselves, as signal-to-noise ratios, have described by Gaussian probabilities (ref. 1).

Most aircraft surveillance radars map the three-dimensional airspace onto a two-dimensional screen called the plan position indicator (PPI). Consequently, there is a resultant loss of information commonly termed collapsing of the coordinates (ref. 2). The range marks on the PPI are measured from the antenna source along the radar beam, which is projected upward at a given angle of elevation. The altitudinal zone covered can be determined by simple trigonometric relationships. The minimum resolution along the beam is related to the pulse width.

There are three several sources of uncertainty using surveillance radars for detection and avoidance. They are the resolution of desired coverage, discrimination between true and false targets which appear on the PPI, along with other system operations uncertainties which are outlined by Richardson (ref. 3) and the behavioral uncertainties of birds flying at specific times and within specified regions. The area of coverage of a beam increases with range, e.g., a 3-degree circular pencil beam elevated 30 degrees from the source above the horizon is 105 feet in diameter at a range of 2000 feet and at a diameter of 314 feet at a range of 6000 feet (ref. 4). Most surveillance radars employ beams which are not circular but have much more sophisticated geometries. Such is the cosecant-squared beam where the echo power from targets of constant radar cross section at constant altitudes is independent of the target's range. This allows for additional coverage of high-altitude targets close to the radar. Geometrical alterations in the radar beams are generally made to overcome blind spots encountered with simple fan beams. The radar energy of a fan beam is diffused as the range increases, as described by the radar equation. At a given range r , an increase in gain, by narrowing the beam angle, increases the signal strength and decreases the effective diameter of the beam at point r .

Flock (ref. 5) has elaborated somewhat on the use of radar to detect bird movements along the arctic coast. He noted some problems following targets over a series of photographic frames made from the PPI. The suitability of radar for determining numbers of birds has been discussed by Gauthreaux (ref. 6) and Nisbet (ref. 7). This involves essentially techniques where radar range or attenuation settings are calibrated against bird counts taken against the silhouette of the moon or within a ceilometer beam.

Richardson (ref. 3) has evaluated the effects of a number of adjustments available on radars and their effect on migration density measurements. These adjustments include Sensitivity Time Control (STC), Circular Polarization, Fast Time Constant (FTC), Instantaneous Automatic Gain Control (IAGC), Constant False Alarm Rate (CFAR), IF gain, and coherent moving target indicator (MTI). He indicates that if reliable quantitative information about bird migration is to be obtained from a surveillance radar, the behavior and mode of that individual radar must be known. With CSC² beams, he states that density estimates must be made between 5 and 30 nmi to avoid biases produced by various heights of flight.

Hunt (ref. 8) concludes that bird density measurements can lead to bird strike probabilities but recommends the use of calibrated variable attenuators.

The development of probabilities has so far been a source of considerable difficulty in the literature. Kohl (ref. 9) began one of the earliest discussions of the use of probabilities to describe the Air Force data bank on bird strike accidents. His approach considers the calculation of the most probable number of nautical miles per bird strike from simple unit analysis and by use of the Poisson density function. The exact rationale leading to the choice of this particular function is unclear since there are many probability functions that might have been selected. Kohl also considers the frontal area as a major parameter in the calculation of strike bird densities.

$$C = \frac{NH_z}{D_p F} \quad (1)$$

where

C = strike bird density

N = number of feet in a nautical mile

D_p = the probable number of nautical miles in which one bird strike can be expected

H_z = vertical distance in feet across a designated altitude zone

F = frontal area of aircraft in square feet

Kohl concludes that airspeed has no effect on the bird strike probability. Since reporting of bird strike accidents sometimes lacks essential information and to bring values of equation 1 in line with actual values, Kohl proposes the use of degradation factors to account for the inadequacy of the data.

Hunt (ref. 10) also considers a similar approach but derives a formula for the bird strike probability per nautical mile flown by considering an arbitrary volume of airspace and defining a number of discrete flight paths determined by the frontal area and geometry of the aircraft. He then sums the individual probabilities calculated over the entire route to arrive at combinational probability for the entire flight.

The following basic problems are noted with these approaches:

(1) The models assume an undefined region of airspace that can in the practical sense lead to a multiplicity of probability values which may become confused and unrelated.

(2) The frontal area parameter is, a priori, a more likely aspect connected with aircraft damage than with bird strike probabilities. The verification of

probabilities of bird(s) residing within the same airspace of an aircraft appears to be a more elementary approach. This is a volumetric consideration.

(3) Summation of individual probabilities with these models over an entire route could result in values greater than unity, which is not allowed.

(4) An assumption of a uniform bird population density over an arbitrary three-dimensional airspace (ref. 10) is not a serious postulate nor can it be considered as an assumption leading to a proper definition of a probability. If one knows this type of configuration with great certainty, then one can fly through vacant cells in sparse bird densities and never hit a bird but could expect to hit a number of birds in less sparse bird densities.

Any fundamental probability model must adhere closely to the following precepts of mathematical modeling.

(a) The model components must have physical or biological significance; that is, there must be a one-to-one correspondence between model component and its physical or biological counterpart.

(b) These components should be measurable in experiments.

In probability models, strict delineation must be discerned between certain events and uncertain events. Such a model is derived from more fundamental considerations and then experimentally verified.

SECTION III

BASIC CONSIDERATIONS

In order to continue in the development of a probability model, it will be useful to review some of the basic definitions and principal properties of probabilities. These follow discussions by Clark and Disney (ref. 11). Korn and Korn (ref. 12) define mathematical probabilities as follows:

"Mathematical probabilities are values of a real numerical function defined on a class of idealized events, which represent results of an experiment or observation."

Given, A , as a set of all possible outcomes of an experiment, such a set is uniquely specified if its elements, a , are known. An inclusion rule is used to specify such a set, as for example:

$$A = \{a: a_1 < a < a_2\} \quad (2)$$

A is a set (equation 2) of all elements, a , which reside between pre-determined values a_1 and a_2 . There exist rules or properties that are characteristic to sets, such as unions, intersections, and complements, which will be reviewed later.

If A_n equals the number of occurrences of event A in the first n repetitions, the probability of A written as $\text{Pr}[A]$ is defined as:

$$\text{Pr}[A] = \lim_{n \rightarrow \infty} \frac{A_n}{n} \quad (3)$$

As the number of repetitions approaches infinity, the limit of the ratio of occurrence to the total number of possible events converges to a value which is defined as a probability. A probability is then a set of real numbers $\text{Pr} = \{p: 0 \leq p \leq 1\}$.

Next, proceed to review properties of probabilities. To accomplish this, a few elementary operations of set theory must be understood. The standard symbolism used in reference 11 will be followed.

The first operation is that of a union. If A and B are both sets, the union of A and B symbolically written $A \cup B$ is a set, whose elements belong to either A or B . (The resulting set is larger than either A or B unless B is a

subset of A, whereas $A \cup B$ would be A). Another operation is that of intersection. The intersection of A and B, symbolically $A \cap B$, is a set whose elements belong to both A and B (the resulting set is smaller than A or B, unless B is a subset of A whereas $A \cap B$ would be B).

The following elementary properties are considered next.

- (a) For any given event A: $\Pr[\bar{A}] = 1 - \Pr[A]$

The probability of A not occurring is simply unity minus the probability of A occurring.

- (b) If \emptyset is an impossible event; then: $\Pr[\emptyset] = 0$

(c) If A_1 and A_2 are any events, not necessarily mutually exclusive, (if A_1 and A_2 are mutually exclusive, they do not occur simultaneously) then the following holds:

$$\Pr [A_1 \cup A_2] = \Pr[A_1] + \Pr[A_2] - \Pr[A_1 \cap A_2]$$

The probability of the union of two sets of events includes the sum of their individual probabilities minus the probability of events which overlap in both sets. The probability of the union of two or more mutually exclusive events is just the sum of their individual probabilities ($A_1 \cap A_2 = \emptyset$).

(d) If one has several events A_1, A_2, \dots, A_k , then the union of these events is given as:

$$\Pr \left[\bigcup_{i=1}^k A_i \right] = \sum_{i=1}^k \Pr[A_i] - \sum_{i < j} \Pr[A_i \cap A_j]$$

One has the sum of their individual probabilities minus the intersection of pairs of events, triples of events, and so forth.

To conclude this section, it is sufficient to make the following statements:

(1) An inclusion rule is defined which closely matches the salient features of a real problem; and (2) given such a rule, it is possible using rational arguments to arrive at more general statements concerning the problem.

A basic probability model for bird-aircraft collisions is presented in section IV.

SECTION IV

DEVELOPMENT OF THE BASIC PROBABILITY MODEL

Figure 1 shows a fan beam for a search radar. The antenna rotates counter-clockwise in a simple scan with a uniform angular velocity sweeping out a series of resolution volumes denoted as V_{B1} , V_{B2} , V_{B3} , ..., V_{BM} , where j is the sequential subscript for each event. Each volume V_{Bj} is defined by a slice of airspace that is or will be occupied by an aircraft according to the radar pulse width, range r , bearing θ , and by the elevational and azimuthal widths of the beam. The volumes V_{pj} are incremental discrete slices of airspace swept out by the aircraft which lie within each V_{Bj} . The geometry of V_{pj} is a solid whose mid-area shape is the frontal normal projection of the aircraft and outer boundaries restricted by V_{Bj} . In such an analysis, any of the exposed aircraft components such as the vertical stabilizer, a fuel tank, a canopy, a wing, or an engine might also be considered. Thus, the resultant probabilities refer to that part and its volume V_{pj} of airspace swept out. Likewise, one would also consider any of its local airspace permeated by radar energy. Also, within the local airspace are birds which show up on the PPI display and whose numbers have been determined by previously mentioned methods.

If one considers any one of these cells such that V_{pj} resides within V_{Bj} and birds are determined to also reside within V_{Bj} , then these birds will either be located in either $V_{Bj} - V_{pj}$, V_{pj} , or both. If as a simplifying assumption, a bird is given an equal chance of being found anywhere within V_{Bj} , the a priori probability, p , of a bird appearing anywhere within V_{pj} is determined only by the size of the volume or cell and is given as*

$$p = \frac{V_{pj}}{V_{Bj}} \quad (4a)$$

Then according to the property (a) listed in section III, the probability of not finding a bird within V_{pj} but within $V_{Bj} - V_{pj}$ is $1-p$ or given as

*This will hold only for small airspaces; for larger airspaces the behavioral distribution of migrating and locally flying birds must be initially considered.

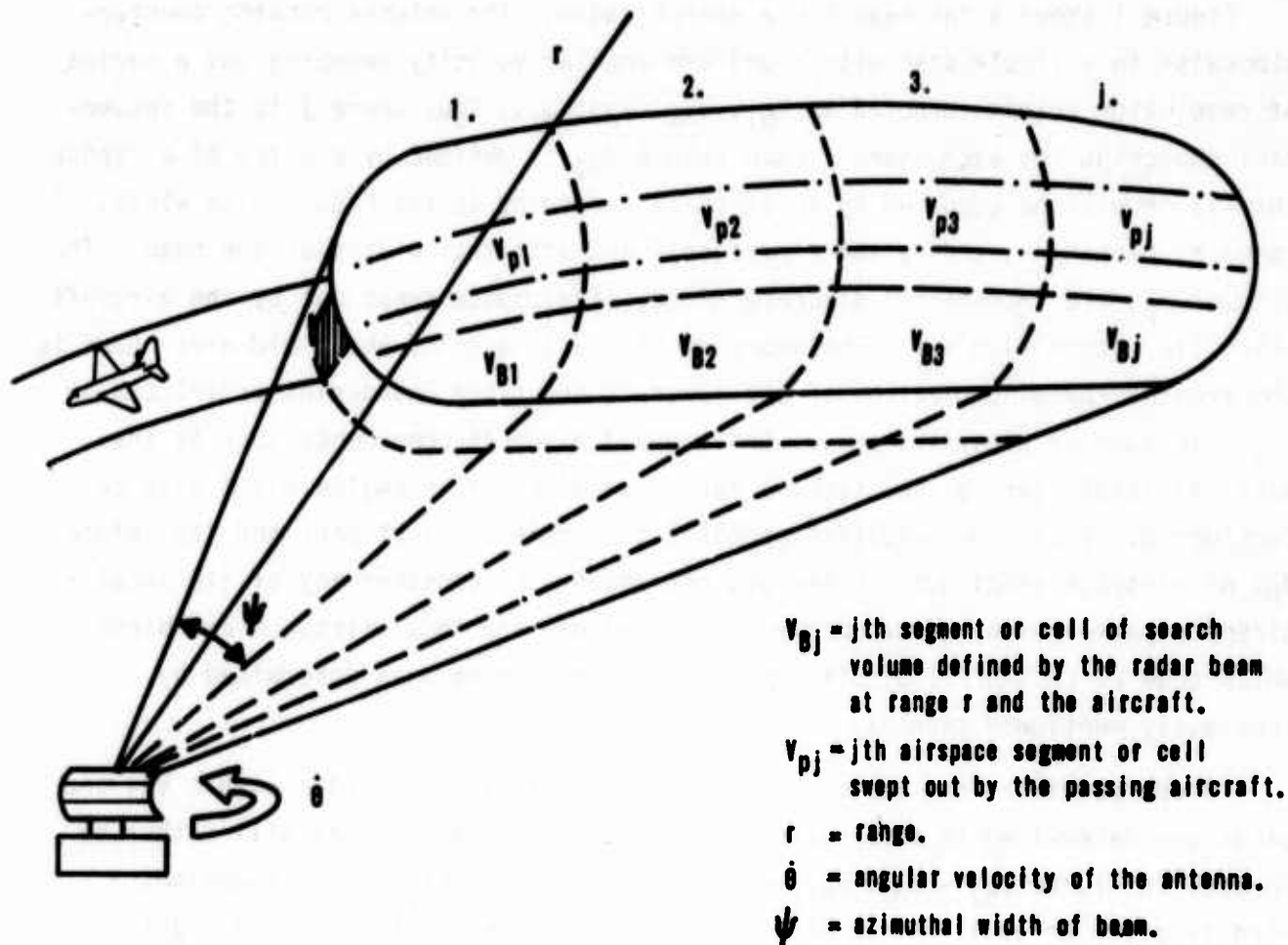


Figure 1. Angular Sweep of Fan Beam with a Passing Aircraft.

$$\bar{p} = \frac{V_{Bj} - V_{pj}}{V_{Bj}} \quad (4b)$$

The probability of three birds then appearing in V_{pj} is p^3 , if there is no logical connection between the occurrence of one or the others; that is, the occurrence of any one bird is an independent event. If n birds are found within V_{Bj} , k of which are found within V_{pj} , of which the probability of those in V_{pj} is p^k and those in $V_{Bj} - V_{pj}$ is $(1-p)^{n-k}$, the probability of this arrangement of birds is $p^k(1-p)^{n-k}$. The selection process of such an event can be accomplished in $\binom{n}{k}$ ways.* Hence, the probability of a total of n birds, k birds found within V_{pj} , is given as:

$$Pr_j(x=k) = \binom{n}{k} p^k (1-p)^{n-k} \quad (5)$$

where x is the discrete random variable for the selection of birds in the j^{th} cell.

Equation 5 is commonly known as the binomial density function. Values of this function are commonly found in standard mathematical tables. For a very large number of possible occurrences, n , Poisson has shown that the following relationship holds.

$$\lim_{n \rightarrow \infty} \binom{n}{k} p^k (1-p)^{n-k} \cong \frac{e^{-np} np^k}{k!} \quad (6)$$

When one investigates tables of binomial probabilities, it is noted that probability values are computed using equation 5 for specific values of n , k , and p . The sum of all probabilities for given values of n and p is unity.

$$\sum_k^n Pr(x=k) = 1 \quad (7)$$

Figure 2 shows a typical binomial distribution plot. That event having the highest probability is usually of greatest interest. Consequently, an investigation of distribution function to determine that event and its corresponding value of k must be accomplished.

* $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

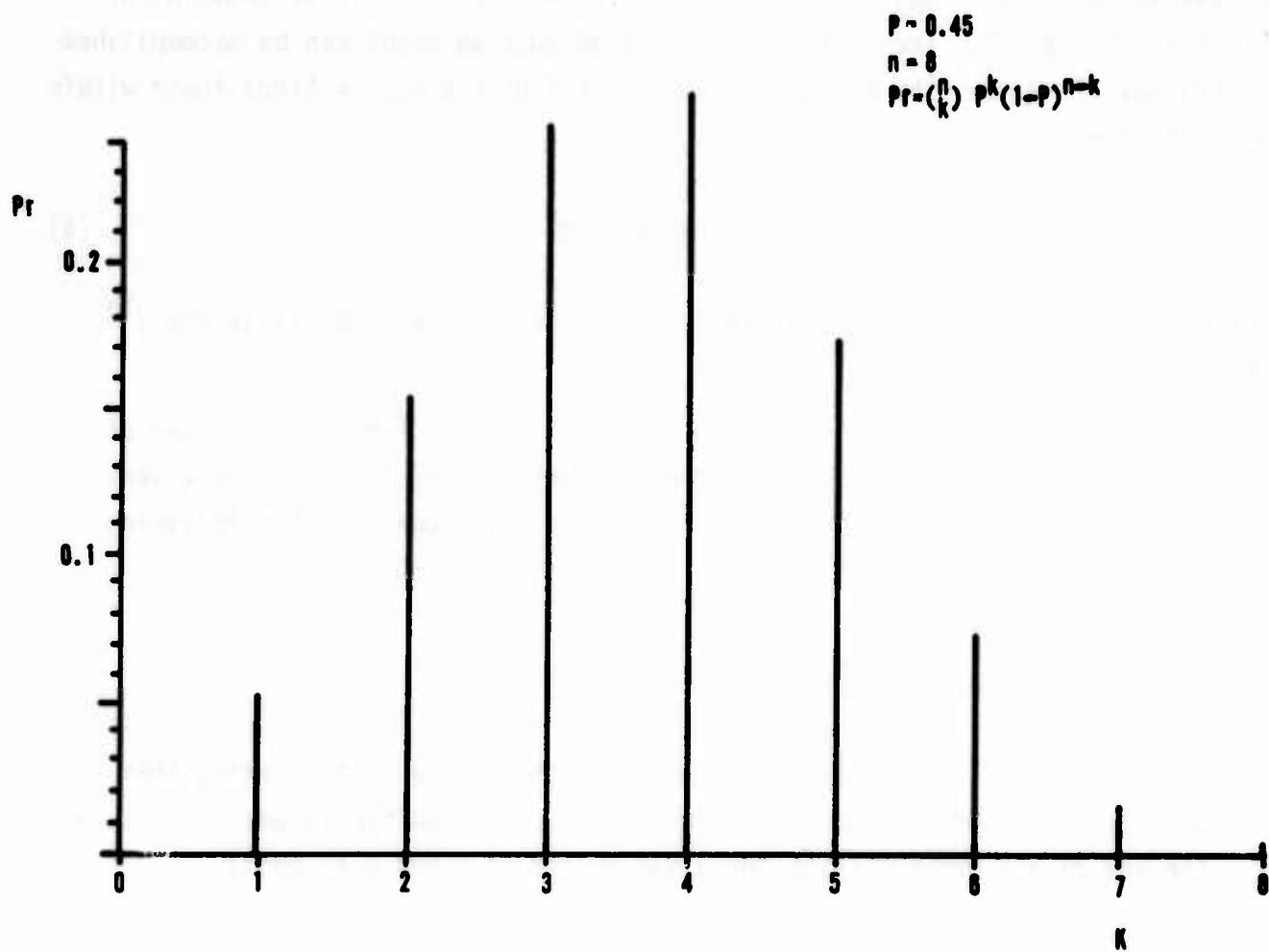


Figure 2. Binomial Probability Distribution Plot.

The next step is to consider the combinational probability as the aircraft moves through each airspace, each with a given number of birds determined at the PPI. Because a continuous space has been approximated by a number of discrete cells, the moving aircraft could at some given time occupy part of any two adjacent cells. Therefore, the events of two adjacent cells could conceivably at some point in time occur simultaneously, thus not being mutually exclusive and exhaustive. The probabilities of each cell, however, are independent; that is to say, that if one considers the conditional probability of the events that occur in any two successive cells, there is no way of determining outcome of one event, knowing the outcome of previous events.* This is represented by the following symbolism.

$$\Pr[A/B] = \Pr[A] \quad (8)$$

where $\Pr[A/B]$ is a conditional probability that A will occur if B is given. The combinational probability for a number of contributing probability sets is the union of those sets. The union of any two sequential cells 1 and 2 is given according to property d given in section III.

$$\Pr[A_1 \cup A_2] = \Pr[A_1] + \Pr[A_2] - \Pr[A_1 \cap A_2] \quad (9)$$

where

$$A_1 = (x=k_1)$$

$$A_2 = (x=k_2)$$

Since A_1 and A_2 are independent events,

$$\Pr[A_1 \cup A_2] = \Pr[A_1] + \Pr[A_2] - \Pr[A_1] \times \Pr[A_2] \quad (10)$$

For the union of all events along the path of flight, the following formula is given:

$$\Pr[A_1 \cup A_2 \cup A_3 \dots \cup A_m] = \sum_{j=1}^m \Pr[A_j] - \sum_{j<l}^m \Pr[A_j] \times \Pr[A_l]$$

where j and l are pairs of events.

*Other arguments similar to these can be given if one were to choose infinitesimal cell sizes and use a continuous approximation for a bird population function. The occurrence of simultaneous events for a finite size aircraft results in events that are independent but not mutually exclusive.

SECTION V

DISCUSSION AND CONCLUSIONS

It has been previously suggested in the literature that general bird strike data can be most closely fitted by the Poisson distribution. However, it must be remembered that Poisson's distribution applies only for the case of a large number of possible occurrences (which may be attributed to too great a study region in such a case). Present USAF bird strike data are generally too broad and meaningless for detection and avoidance decisions at the base level. Data on localized bird concentrations along specific air routes are not presently available to test the given binomial probability equations. This data should be collected in the future by radar systems of well known response and properly calibrated for bird census measurements. It should provide multi-dimensional information (geographical coordinates, altitude, type of birds, numbers, direction, and velocity of movement).

The probability analysis given in this report begins from an elementary probability assumption (none more elementary can be made in this case). Such analysis began either from this equal-chance assumption or from elementary experimental probabilities (mean accident frequency data). Elementary probability data could be obtained if future bird-strike data collection could meet the criteria suggested in the previous paragraph. These data will have to be taken over a period of several years. Moreover, such precise accident recording may not always be feasible without special sensing equipment on board the aircraft. This may be beyond the scope of present bird-strike recording.

The presentation given in this report is a fundamental insight into the complex problem of risk analysis. With additional work, this model can be the basis of future solutions to the USAF bird-aircraft strike problem. The model must be expanded to include the geometry of defined airspaces for specific air routes. It must include the geometric and physical characteristics of specific radar systems employed, beam shapes, aircraft geometrics, closing-in directions and velocities (migratory phenomena, etc.), and then field tested. This model is then simulated and tested to produce solutions from which 'hazard' tables, sample graphs, etc., are constructed that can be easily used by the Air Traffic Controller or by the pilots. These devices will aid them to quickly, accurately,

and efficiently determine impending hazards and what they mean. Only this approach will be taken with great confidence, since it is based on very fundamental considerations.

In summary, recommendations for future research include the following:

(1) A computer simulation should be conducted using an advanced probability model that includes the geometry and physical characteristics of current radar systems, beam shapes, aircraft types, etc. Optimal scanning patterns should be investigated for rapid assessment of impending bird hazards. Field studies should be conducted, data of which will be used to realign the basic model and to determine the direction of future research leading to the solution and understanding of bird-strike problems.

(2) Rationale and presentation of data for decision-making based on probability analysis needs to be accomplished for preflight preparation and during flight when bird hazards occur.

(3) Basic research should continue to consider alternate feasible methods for rapid bird detection (preferably automated types that will not increase existing pilot or ATC workloads).

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